OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. The total number of factors of a prime number is:
(a) 1 (b) 0
(c) 2 (d) 3

Ans. *(c) 2*
Explanation: Factors of a prime number are 1 and the number itself.

2. If a and b are two co-prime numbers, then a^3 and b^3 are:
(a) co-prime (b) not co-prime
(c) even (d) odd
[CBSE Term-1 Std. 2021]

Ans. *(b)* xy^2
Explanation:
Given that, $a = x^3y^2 = x \times x \times x \times y \times y$
and $b = xy^3 = x \times y \times y \times y$
We know that HCF of two or more numbers is the product of the smallest power of each common prime factor involved in the numbers.
$\therefore$ HCF of a and b = HCF (x^3y^2, xy^3)
$= x \times y \times y = xy^2$

5. $(7 \times 11 \times 13 \times 15) + 15$ is a:
(a) composite number
(b) whole number
(c) prime number
(d) both (a) and (b)

Ans. *(d) both (a) and (b)*
Explanation: $(7 \times 11 \times 13 \times 15) + 15$
$= 15 (7 \times 11 \times 13 + 1)$
$= 15 \times 1002$
Clearly, 15×1002 is a whole number.
Also, number is having more than two factors.
Therefore, it is a composite number as well as a whole number.

6. The LCM of two numbers is 2400. Which of the following cannot be their HCF ?
(a) 300 (b) 400
(c) 500 (d) 600
[CBSE Term-1 Std. 2021]

7. If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a equals to:
(a) 1 (b) 2
(c) 3 (d) 4

Ans. *(d) 4*
Explanation: We know that,
LCM $(a, b) \times$ HCF $(a, b) = a \times b$
$\Rightarrow \quad 36 \times 2 = a \times 18$
$\Rightarrow \quad a = \frac{36 \times 2}{18}$
$\Rightarrow \quad a = 4$

8. The product of a non-zero rational and an irrational number is:
(a) always irrational
(b) always rational
(c) rational or irrational
(d) one [NCERT Exemplar]

Ans. *(a) always irrational*
Explanation: Product of a non-zero rational and an irrational number is always irrational.
For example:
$\frac{7}{9}$ is rational and $\sqrt{2}$ is an irrational number.
Then,
$\frac{7}{9} \times \sqrt{2} = \frac{7\sqrt{2}}{9}$, which is an irrational number.

Ans. *(a) co-prime*
Explanation: Co-prime numbers are the set of numbers that don't have any common factor other than 1. Here, a^3 are b^3 are co-prime.

3. The LCM of smallest two-digit composite number and smallest composite number is:
(a) 12 (b) 4
(c) 20 (d) 44 [CBSE SQP 2019]

4. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x and y are prime numbers, then the HCF (a, b) is:
(a) xy (b) xy^2
(c) x^3y^3 (d) x^2y^2 [Diksha]

9. The greatest number which when divides 1251, 9377 and 15628 leaves remainder 1, 2 and 3 respectively is:
(a) 575 (b) 450
(c) 750 (d) 625
[CBSE Term-1 Std. 2021]

Ans. *(d) 625*
Explanation: Required greatest number
HCF $(1251 - 1, 9377 - 2, 15628 - 3)$
$=$ HCF $(1250, 9375, 15625)$
$\because \quad 1250 = 2 \times 5^4$
$9375 = 3 \times 5^5$
$15625 = 5^6$
HCF $(1250, 9375, 15625) = 5^4 = 625$
Hence, the required greatest number is 625.

10. If HCF of 144 and 180 is expressed in the form $13m - 3$, then m is:
(a) 1 (b) 2
(c) 3 (d) 4 [Diksha]

11. Three alarm clocks ring their alarms at regular intervals of 20 min, 25 min and 30 min respectively. If they first beep together at 12 noon, at what time will they beep again for the first time ?
(a) 4 : 00 pm (b) 4 : 30 pm
(c) 5 : 00 pm (d) 5 : 30 pm
[CBSE Term-1 Std. 2021]

Ans. *(c) 5:00 pm*
Explanation: Minutes after which they beep together = LCM (20, 25, 30)
$\because \quad 20 = 2^2 \times 5$
$25 = 5^2$
$30 = 2 \times 3 \times 5$
$\therefore \quad$ LCM $(20, 25, 30) = 2^2 \times 3 \times 5^2$
$= 300$ min
$= 5$ hours
Now, three alarm clocks ring their alarms at 12 noon, so next they will beep together after 5 hours *i.e.*, at 5:00 pm.

12. If HCF $(a, b) = 45$ and $a \times b = 30375$, then LCM (a, b) is:
(a) 1875 (b) 1350
(c) 625 (d) 675

13. **The ratio of LCM and HCF of the least composite and the least prime number is:**

(a) 1 : 2 (b) 2 : 1

(c) 1 : 1 (d) 1 : 3

[CBSE Term-1 SQP 2021]

Ans. *(b) 2 : 1*

Least composite number is 4 and the least prime number is 2

LCM(4, 2): HCF (4, 2) = 4 : 2 = 2 : 1

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: Least composite number = $4 = 2^2$

and, Least prime number = 2

$\therefore$ HCF (4, 2) = 2

and LCM (4, 2) = 4

$\therefore \quad \frac{\text{LCM}(4,2)}{\text{HCF}(4,2)} = \frac{4}{2} = \frac{2}{1}$

14. **The LCM of two prime numbers p and q ($p > q$) is 221. Find the value of $3p - q$.**

(a) 4 (b) 28

(c) 38 (d) 48

[CBSE Term-1 SQP 2021]

Ans. *(c) 38*

LCM of two prime numbers = product of the numbers

$221 = 13 \times 17$

So $p = 17$ and $q = 13$

$\therefore \quad 3p - q = 51 - 13 = 38$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: The numbers p and q are prime numbers,

$\therefore$ HCF $(p, q) = 1$

Here, LCM$(p, q) = 221$

As, $p > q$

$\therefore \quad p = 17, q = 13$ (As $p \times q = 221$)

Now, $3p - q = 3 \times 17 - 13$

$= 51 - 13$

$= 38$

15. **If HCF of two numbers is 1, the numbers are called relatively ..... or ..... .**

(a) prime, co-prime (b) composite, prime

(c) both (a) and (b) (d) none of these

[Diksha]

16. **The exponent of 5 in the prime factorisation of 3750 is:**

(a) 3 (b) 4

(c) 5 (d) 6

[CBSE Term-1 Std. 2021]

Ans. *(b) 4*

Explanation: We have,

5	3750
5	750
5	150
5	30
3	6
2	2
	1

$3750 = 2 \times 3 \times 5^4$

So, the exponent of 5 in the prime factorisation of 3750 is 4.

17. **Three bells ring at intervals of 48, 60 and 72 mins. If they start ringing together after how much time will they next ring together?**

(a) 540 mins (b) 1440 mins

(c) 320 mins (d) 720 mins

[Diksha]

Ans. *(d) 720 mins*

Explanation: Here, we need to find the LCM of the numbers 48, 60, 72.

By prime factorisation,

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

$72 = 2^3 \times 3^2$

$\text{LCM} = 2^4 \times 3^2 \times 5$

$= 720$ mins

Therefore, the three bells ring after 720 mins.

18. **The values of x and y in the figure are:**

(a) $x = 10; y = 14$ (b) $x = 21; y = 84$

(c) $x = 21; y = 25$ (d) $x = 10; y = 40$

[CBSE 2013]

19. **Let a and b be two positive integers such that $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers. If HCF$(a, b) = p^mq^n$ and LCM$(a, b) = p^rq^s$, then $(m + n)(r + s) =$**

(a) 15 (b) 30

(c) 35 (d) 72

[CBSE SQP Std. 2022]

Ans. *(c) 35*
[CBSE Marking Scheme SQP Std. 2022]

Explanation: Given,

$a = p^3q^4$

and $b = p^2q^3$

$\therefore$ HCF of $(ab) = p^2q^3$

$p^mq^n = p^2q^3$

$\Rightarrow m = 2$

$\Rightarrow n = 3$

LCM of $(a, b) = p^3q^4$

$p^rq^s = p^3q^4$

$\Rightarrow r = 3$

$\Rightarrow s = 4$

$(m + n)(r + s) = (2 + 3)(3 + 4)$

$\Rightarrow = 5 \times 7$

$= 35.$

20. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM (p, q) is:

(a) ab **(b) a^2b^2**
(c) a^3b^2 **(d) a^3b^3**

[CBSE SQP Basic 2022, NCERT Exemplar]

Ans. *(c) a^3b^2*
[CBSE Marking Scheme SQP Basic 2022]

Explanation: Given that, $p = ab^2 = a \times b \times b$

And $q = a^3b = a \times a \times a \times b$

LCM of p and q = LCM (ab^2, a^3b)

$= a \times b \times b \times a \times a$

$= a^3b^2$.

[Since, LCM is the product of the greatest power of each prime factor involved in the numbers]

Caution

Students usually make mistakes while calculating prime factors. They should always start with the lowest prime number.

21. There are two batches of students in a college. The first batch has 135 students whereas the second batch has 225 students. These students have to be seated in an auditorium in such a way that each row has the same number of students of each batch.

The greatest number of seats in each row should be:

(a) 15 **(b) 75**
(c) 45 **(d) 5**

Ans. *(c) 45*

Explanation: Here,

$135 = 3 \times 3 \times 3 \times 5 = 3^3 \times 5^1$

and $225 = 3 \times 3 \times 5 \times 5 = 3^2 \times 5^2$

So, HCF $(135, 225) = 3^2 \times 5^1$ *i.e.*, 45

22. Rahul has 40 cm long red and 84 cm long blue ribbon. He cuts each ribbon into pieces such that all pieces are of equal length. What is the length of each piece?

(a) 4 cm **(b) 5 cm**
(c) 6 cm **(d) 8 cm**

[CBSE Question Bank 2022]

Ans. *(a) 4 cm*

Explanation: Prime factor of $40 = 2 \times 2 \times 2 \times 5$

Prime factor of $84 = 2 \times 2 \times 3 \times 7$

Now, HCF of 40 and $84 = 2 \times 2$

$= 4$

Thus, the length of each piece is 4 cm.

23. While coming from the school, Virat noticed that the particulate matter $PM_{2.5}$ levels were exceptionally high as he had difficulty in breathing. On reaching home, he checked the reports and found that city's Air Quality Index was 196, which was "poor" by WHO standards.

As Virat loved to play with numbers, he decided to find the prime factors of 196 and also to evaluate the sum of exponents of prime factors of this number.

The sum of exponents of prime factors in the prime factorisation of 196 is:

(a) 3 **(b) 4**
(c) 5 **(d) 6** **[CBSE 2020]**

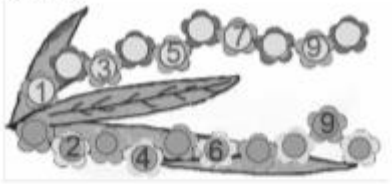
Ans. *(b) 4*

Explanation: The prime factorisation of 196 is:

$196 = 2^2 \times 7^2$

So, sum of the exponents of prime factors 2 and 7 is $2 + 2$ *i.e.*, 4.

24. Suraj was teaching counting from 1 to 10 to his sister who is a kindergarten student. Suddenly, a question came to his mind- Are there any numbers that are divisible by all the numbers from 1 to 10 and if yes, how to find them?



The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is:

(a) 10 (b) 100

(c) 504 (d) 2520

[NCERT Exemplar]

Ans. *(d) 2520*

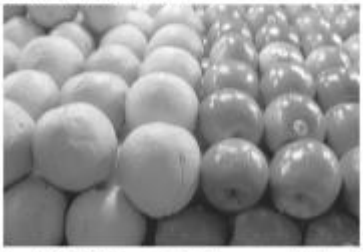
Explanation: As we require the least number, the problem is based on finding the LCM.

Factors of 1 to 10 numbers are as follows:

$1 = 1$
$2 = 1 \times 2$
$3 = 1 \times 3$
$4 = 1 \times 2 \times 2$
$5 = 1 \times 5$
$6 = 1 \times 2 \times 3$
$7 = 1 \times 7$
$8 = 1 \times 2 \times 2 \times 2$
$9 = 1 \times 3 \times 3$
$10 = 1 \times 2 \times 5$

LCM of numbers 1 to 10
$= \text{LCM}(1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$
$= 1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$
$= 2520$

25. Palak had 65 oranges and 117 apples which she wanted to pack in separate packets in such a way that there would be equal number of oranges or apples in each packet. So, she figure out that she needed to find the HCF of 65 and 117.



If the HCF of 65 and 117 is expressible in the form $65m - 117$, then the value of m is:

(a) 4 (b) 2

(c) 1 (d) 3

[CBSE 2014, 12, NCERT Exemplar]

Fill in the Blanks

26. $\left(\frac{2+\sqrt{5}}{3}\right)$ is a/an ..... number.

[CBSE 2020]

Ans. irrational

Explanation: As $\sqrt{5}$ is irrational, so $2+\sqrt{5}$ is irrational and hence $\frac{2+\sqrt{5}}{3}$ is an irrational number.

27. The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, the other number is ..... .

28. If $a = (2^2 \times 3^3 \times 5^4)$ and $b = (2^3 \times 3^2 \times 5)$ then HCF (a, b) is ..... .

Ans. 180

Explanation:
$a = (2^2 \times 3^3 \times 5^4)$
$b = (2^3 \times 3^2 \times 5)$
$\text{HCF}(a, b) = 2^2 \times 3^2 \times 5$
$= 4 \times 9 \times 5 = 180$

29. Product of two numbers is 18144 and their HCF is 6, then their LCM is ..... .

Ans. 3024

Explanation:
We know,
Product of two numbers
$= \text{HCF} \times \text{LCM} = 18144$
$\Rightarrow \quad 6 \times \text{LCM} = 18144$
$\Rightarrow \quad \text{LCM} = \frac{18144}{6} = 3024$

30. The HCF of smallest composite number and the smallest prime number is ..... .

Ans. 2

Explanation: Smallest prime number = 2
Smallest composite number = 4
$\therefore \quad \text{HCF}(2, 4) = 2$

31. If a and b are positive integers, then $\frac{\text{HCF}(a,b) \times \text{LCM}(a,b)}{ab} =$ .....

32. ..... is the HCF of two consecutive even numbers.

Assertion Reason

Direction for questions 33 to 38: In question number 33 to 38, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) **Assertion (A) is true but reason (R) is false.**

(d) **Assertion (A) is false but reason (R) is true.**

33. **Assertion (A): If product of two numbers is 5780 and their HCF is 17, then their LCM is 340.**

Reason (R): HCF is always a factor of LCM

[CBSE SQP Std. 2022]

Ans. *(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)*

[CBSE Marking Scheme SQP Std. 2022]

Explanation: We know that,

$$\text{HCF} \times \text{LCM} = \text{Product of the numbers}$$

$$\Rightarrow \quad 17 \times \text{LCM} = 5780$$

$$\Rightarrow \quad \text{LCM} = \frac{5780}{17}$$

$$\Rightarrow \quad \text{LCM} = 340$$

The LCM of the two numbers = 340

HCF of two numbers is always a factor of their LCM.

Hence, both assertion and reason are true but reason is not the correct explanation of assertion.

34. **Assertion (A): The HCF of two numbers is 6 and their product is 300, then their LCM is 50.**

Reason (R): For any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

Ans. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).*

Explanation: We know that,

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

$$\text{LCM} \times 6 = 300$$

$$\text{LCM} = \frac{300}{6} = 50$$

Hence, both assertion and reason are true and the reason is the correct explanation of the assertion.

35. **Assertion (A): 5 is a rational number.**

Reason (R): The square roots of all positive integers are irrational.

Ans. *(c) Assertion (A) is true but reason (R) is false.*

Explanation: As, 5 is a rational number, but square root of all positive integers are not necessarily irrational.

For example, $\sqrt{9} = 3$ which is rational.

Hence, assertion is true but the reason is false.

36. **Assertion (A): The product of $(3 + \sqrt{3})$ and $(2 - \sqrt{3})$ is an irrational number.**

Reason (R): The product of two irrational numbers is always an irrational number.

37. **Assertion (A): $\sqrt{2}$ is an irrational number.**

Reason (R): If m is a natural number which is not a perfect square, then $\sqrt{m}$ is irrational.

Ans. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).*

Explanation: We know that, if m is a natural number which is not a perfect square, then $\sqrt{m}$ is irrational.

2 is a natural number and not a perfect square.

Thus, $\sqrt{2}$ is an irrational number.

Hence, both assertion and reason are true and the reason is the correct explanation of the assertion.

38. **Assertion (A): The prime factors of 3750 are 2 × 3 × 54.**

Reason (R): Composite numbers are the numbers which have atleast one factor other than 1 and the number itself.

Ans. *(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).*

Explanation: The prime factors of 3750 $= 2 \times 3 \times 5^4$.

We know that, the numbers which have atleast one factor other than 1 and the number itself are called composite numbers.

Hence, both assertion and reason are true but the reason is not the correct explanation of the assertion.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

39. *The school principal wants to address the students of classes eighth to tenth on the importance of personal hygiene. He asks the teacher in charge of the school to arrange all the students of eighth, ninth and tenth classes in a single hall. There are 84 students from eighth class, 63 students from ninth and 42 students from tenth class.*

(A) **What is the minimum number of rows in which the students can be seated such that students belonging to the same class are seated in the same row?**

(a) 7 (b) 9
(c) 21 (d) 42

(B) **Realising that the hall can accommodate more students, the school Principal now wants to include 98 students of class seventh also. What will be the number of students and minimum number of rows such that students belonging to the same class are seated in the same row?**

	Number of students of same class in one row	Minimum number of rows
(a)	14	21
(b)	21	14
(c)	41	7
(d)	7	41

(C) **Suppose the bus incharge of the school has to arrange buses in the morning for a picnic. There are two lines of buses, line A and line B. Buses on line A leave after every 15 minutes while buses on line B leave after every 20 minutes. In a day, how many times do buses on both lines A and B leave together between 8 a.m. and 11 a.m, if firstly, buses leave together at 8 a.m.?**

(a) 3 (b) 4
(c) 5 (d) 6

(D) **Three numbers are in the ratio of 3 : 4 : 5 and their LCM is 2400. The HCF of the numbers is:**

(a) 40 (b) 60
(c) 80 (d) 120

(E) **The product of two numbers is 2028 and their HCF is 13. The LCM of the numbers is:**

(a) 13 (b) 156
(c) 2028 (d) 26364

Ans. (A) *(b) 9*

Explanation: As we have to find the minimum number of rows, we have to first take the HCF of 84, 63 and 42.

$84 = 2^2 \times 3 \times 7$

$63 = 3^2 \times 7$

$42 = 2 \times 3 \times 7$

$\therefore$ HCF $= 3 \times 7 = 21$

So, 21 students of each class can be seated in one row. That means there will be 4 rows for class eighth, 3 rows for class ninth and 2 rows for class tenth.

Total number of rows = 4 + 3 + 2 = 9

(B)

	Number of students of same class in one row	*Minimum number of rows*
(d)	*7*	*41*

Explanation: We will now calculate the HCF of 42, 63, 84 and 98.

$42 = 2 \times 3 \times 7$

$63 = 3^2 \times 7$

$84 = 2^2 \times 3 \times 7$

$98 = 2 \times 7^2$

$\therefore$ HCF $= 7$

So, 7 students of each class can now be seated in one row. That means there will be 12 rows for class eighth, 9 rows for class ninth, 6 rows for class tenth and 14 rows for class seventh.

Total number of rows = 12 + 9 + 6 + 14 = 41

(D) *(a) 40*

Explanation: Let the numbers be $3x$, $4x$ and $5x$.

Their LCM $= 3 \times 4 \times 5 \times x = 60x = 2400$

$\Rightarrow \quad x = 40.$

Thus, the numbers are 120, 160 and 200.

To find their HCF, we find their prime factors:

$$120 = 2^3 \times 3 \times 5$$
$$160 = 2^5 \times 5$$
$$200 = 2^3 \times 5^2$$
$$\therefore \quad \text{HCF} = 2^3 \times 5 = 40$$

40. *Raman is hosting the new year party for his friends. He wants to purchase some eatables like patties and buns for making burgers, some towels and some shining papers and glitter papers for decorating his house.*

When he reaches the store, he finds that patties come in a pack of 6, and buns in a pack of 8. Moreover, no small towels are available and the only size available is 16 m × 20 m. The shiny paper comes in lengths of 36 inches and the glitter paper in lengths of 40 inches.

(A) Find the prime factorisation of 16 × 20.

(B) Assuming one patty is required for one bun, what is the smallest number of packs of each, Raman must buy so that no patty or bun is left out?

(C) Square towels has to be cut from the piece of cloth measuring 16 m x 20 m. Find the minimum number of towels that can be cut so that there is no wastage.

Ans. (A) $16 \times 20 = 320$

$$\therefore \quad 320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$$
$$= 2^6 \times 5$$

(B) As each packet of patties contains 6 patties and each packet of buns contains 8 buns, we have to take the LCM of 6 and 8 to find the minimum number of packets of each so that no patty or bun is left out.

Factors of $6 = 2 \times 3$; $8 = 2^3$

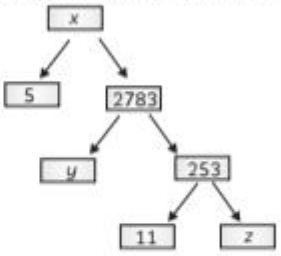
LCM (6, 8) $= 2^3 \times 3 = 24$

Therefore, 24 patties and 24 buns are required.

Number of packets of patties $= \frac{24}{6} = 4$;

Number of packets of buns $= \frac{24}{8} = 3$

41. *A mathematics exhibition is being conducted in your school and one of your friends is making a model of a factor tree. He has some difficulty and asks for your help in completing a quiz for the audience.*



Observe the following factor tree and answer the following questions:

(A) Find the value of *x*.

(B) Find the value of *y* and *z*.

(C) Find the prime factorisation of 13915.

[CBSE Question Bank 2022]

Ans. (A) Here, $x = 5 \times 2783$

$$= 13915$$

(C) $13915 = 5 \times y \times 11 \times z$

$$= 5 \times 11 \times 11 \times 23$$
$$= 5 \times 11^2 \times 23$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

42. State the Fundamental Theorem of Arithmetic. [CBSE SQP 2020]

Ans. Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the factors occur. [CBSE Marking Scheme SQP 2020]

43. Express 156 as the product of primes. [CBSE SQP 2020]

Ans. $156 = 2^2 \times 3 \times 13$ [CBSE Marking Scheme SQP 2020]

Explanation:

Given, number is 156.

2	156
2	78
3	39
13	13
	1

Then, $156 = 2 \times 2 \times 3 \times 13$

$= 2^2 \times 3 \times 13$

Hence, prime factors of 156 are $2^2 \times 3 \times 13$.

44. Given that HCF (96, 404) is 4, find the LCM (96, 404). [CBSE SQP 2020]

45. If $xy = 180$ and HCF$(x, y) = 3$, then find the LCM(x, y). [Delhi Gov. SQP 2022]

46. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other number. [CBSE 2020]

Ans. We know that

HCF $(a, b) \times$ LCM $(a, b) = a \times b$

So, $13 \times 182 = 26 \times b$

$\Rightarrow \quad b = \frac{13 \times 182}{26} = 91$

Thus, the other number is 91.

47. Given that HCF (135, 225) = 45, find the LCM (135, 225). [CBSE 2020]

Ans. We know that

LCM × HCF = Product of two numbers

$\therefore$ LCM (135, 225) $= \frac{\text{Product of 135 and 225}}{\text{HCF}(135, 225)}$

$= \frac{135 \times 225}{45} = 675$

48. Are the smallest prime and the smallest composite numbers co-prime? Justify. [Diksha]

Ans. No, we know that, smallest prime number is 2 and smallest composite number is 4.

And, HCF of (2, 4) = 2

Since, there is a common factor 2 between them. So, they are not co-primes.

49. The HCF of two numbers a and b is 5 and their LCM is 200. Find the product ab. [CBSE 2019]

50. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons. [NCERT Exemplar]

51. Find a rational number between $\sqrt{2}$ and $\sqrt{7}$. [CBSE 2019]

Ans. $\sqrt{2} = 1.414$ and $\sqrt{7} = 2.645$

Let the required rational number be x.

$\therefore \quad \sqrt{2} < x < \sqrt{7}$

or $\quad 1.414 < x < 2.645$

Hence, any rational number like 1.5, 2.0, 2.5, can be the answer.

52. Write the number of zeroes in the end of a number whose prime factorisation is $2^2 \times 5^3 \times 3^2 \times 17$. [CBSE 2019]

Ans. Given, $2^2 \times 5^3 \times 3^2 \times 17$

$= (2 \times 5)^2 \times 5 \times 3^2 \times 17$

$= (10)^2 \times 5 \times 3^2 \times 17$

[$\because$ on multiplying 2×5, we get 10]

The power of 10 in the given expression is 2.

Hence, the number of zeroes in the end will be 2.

53. If the HCF (336, 54) = 6, find the LCM (336, 54). [CBSE 2019]

Ans. The HCF (336, 54) = 6.

We know that,

LCM × HCF = Product of two numbers

$$\Rightarrow \quad \text{LCM} = \frac{336 \times 54}{6}$$

$$= 336 \times 9 = 3024$$

Hence, the LCM of the two numbers is 3024.

54. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$. [CBSE SQP 2019]

55. Write one rational and one irrational number lying between 0.25 and 0.32. [CBSE SQP 2020]

Ans.

Rational number = 0.30

Irrational number = 0.3010203040...

Or any other correct rational and irrational number.

[CBSE Marking Scheme SQP 2020]

56. Vivek has been reading in the newspapers about the imposition of Section 144 of CrPC in different cities in the wake of rising cases of coronavirus.

As he has a mathematical mind, he knew that the number 144 is a perfect square. At the same time, he also wanted to know its prime factors.

Write the exponent of 3 in the prime factorisation of 144. [Diksha]

Ans. Prime factorisation of $144 = 2^4 \times 3^2$

So, exponent of 3 is 2.

57. Vicky went to the stationery shop to buy 45 pencils and 105 pens for giving as return gifts to his friends on his birthday party. He asked the shopkeeper to pack them in such a way that each packet has equal number of pencils or pens. So, he calculated and told the shopkeeper to put 15 pencils or pens in each packet.

The HCF of 45 and 105 is 15. Find their LCM. [CBSE 2010]

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

58. Express 255 as a product of prime factors. [British Council 2022]

Ans. Given, number is 255

3	255
5	85
17	17
	1

Thus, $255 = 3 \times 5 \times 17$

59. The product of the LCM and HCF of two natural numbers is 24. The difference of two numbers is 2. Find the numbers. [Diksha]

60. Two alarm clocks ring their alarms at regular intervals of 72 seconds and 50 seconds, respectively. If they first beep together at 12 noon, at what time will they beep again for the second time? [Diksha]

61. Find the sum of exponents of prime factors in the prime factorisation of 21600. [British Council 2022]

Ans. Given, number is 21600

2	21600
2	10800
2	5400
2	2700
2	1350
3	675
3	225
3	75
5	25
5	5
	1

Thus,

$21600 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$

$= 2^5 \times 3^3 \times 5^2$

Sum of exponents = 5 + 3 + 2 = 10

62. Three bells ring at an interval of 4, 7 and 14 minutes. All three bells rang at 6 am, when the three bells will ring together next? **[CBSE SQP 2020]**

Ans.

$4 = 2 \times 2$

$7 = 7 \times 1$

$14 = 2 \times 7$

$LCM = 2 \times 2 \times 7 = 28$

The three bells will ring together again at 6 : 28 am

[CBSE Marking Scheme SQP 2020]

Explanation: The time after which the three bells will ring together is the LCM of 4, 7 and 14.

Now, $4 = 2 \times 2$

$7 = 7 \times 1$

$14 = 2 \times 7$

$\therefore$ $LCM = 2 \times 2 \times 7 = 28$

Since, the bells rang at 6 am, so they will again ring together at 6 : 28 am.

63. Find the HCF of 612 and 1314 using prime factorisation. **[CBSE 2019]**

64. Write the smallest number which is divisible by both 306 and 657. **[CBSE 2019]**

Ans. Given numbers are 306 and 657.

The smallest number divisible by 306 and 657 = LCM(306, 657)

Prime factors of 306 $= 2 \times 3 \times 3 \times 17$

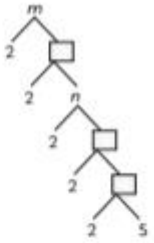
Prime factors of 657 $= 3 \times 3 \times 73$

$\therefore$ LCM of (306, 657) $= 2 \times 3 \times 3 \times 17 \times 73$

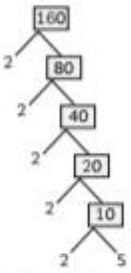
$= 22338$

Hence, the smallest number divisible by 306 and 657 is 22,338.

65. In the adjoining factor tree, find the numbers *m* and *n*.



Ans.



So, $m = 160$ and $n = 40$.

66. Write any two irrational numbers whose product is a rational number.

67. Using prime factorisation method, find the HCF and LCM of 210 and 175. **[CBSE 2011]**

Ans. The prime factorisation of 210 and 175 are:

$210 = 2 \times 3 \times 5 \times 7$

$175 = 5 \times 5 \times 7$

So, HCF (210, 175) $= 5 \times 7 = 35$; and

LCM (210, 175) $= 2 \times 3 \times 5 \times 5 \times 7 = 1050$

68. Find the two numbers which on multiplication with $\sqrt{360}$ gives a rational number. Are these numbers rational or irrational? [Diksha]

Ans. $\sqrt{360} = \sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 5}$

$= 6\sqrt{10}$

If we multiply $6\sqrt{10}$ with $\sqrt{10}$ and 1, we get

$6\sqrt{10} \times \sqrt{10} \times 1 = 60$, which is a rational number.

Hence, required numbers are $\sqrt{10}$ and 1.

Here, 1 is a rational number and $\sqrt{10}$ is an irrational number.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

69. Given that $\sqrt{3}$ is irrational, prove that $5+2\sqrt{3}$ is irrational. [CBSE SQP Std. 2022]

Ans. Let us assume $5+2\sqrt{3}$ is rational, then it must be in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$

i.e. $5+2\sqrt{3} = \frac{p}{q}$

So $\sqrt{3} = \frac{p-5q}{2q}$ ...(i)

Since p, q, 5 and 2 are integers and $q \neq 0$ R,

HS of equation (i) is rational. But LHS of (i) is $\sqrt{3}$ which is irrational.

This is not possible. This contradiction has arisen due to our wrong assumption that $5 + 2\sqrt{3}$ is rational. So, $5+2\sqrt{3}$ is irrational.

[CBSE Marking Scheme SQP Std. 2022]

70. Rakesh is preparing dinner plates. He has 12 pieces of chicken and 16 rotis. If he wants to make all the plates identical without any food left over, what is the greatest number of plates Rakesh can prepare? [Diksha]

71. Prove that $\sqrt{5}$ is an irrational number. [CBSE 2014]

Ans. Let, $\sqrt{5}$ be a rational number and its simplest form is $\frac{a}{b}$, where a and b are integers having no common factor other than 1 and $b \neq 0$.

Now, $\sqrt{5} = \frac{a}{b}$

Squaring both sides, we get

$$5 = \frac{a^2}{b^2}$$

$\Rightarrow 5b^2 = a^2$ ...(i)

$\Rightarrow a^2$ is divisible by 5 [$\because 5b^2$ is divisible by 5]

$\Rightarrow a$ is divisible by 5 [$\because$ 5 is a prime number and divides $a^2 \Rightarrow 5$ divides a]

$\therefore$ Let $a = 5c$, for some integer 'c'.

On substituting $a = 5c$ in equation (i), we get

$5b^2 = (5c)^2$

$\Rightarrow 5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

$\Rightarrow b^2$ is divisible by 5 [$\because 5c^2$ is divisible by 5]

$\Rightarrow b$ is divisible by 5

Since, a and b are both divisible by 5, 5 is common factor of a and b.

But this contradicts the fact that a and b have no common factor other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is a rational number.

Hence, $\sqrt{5}$ is an irrational number.

72. Prove that $2 - \sqrt{3}$ is irrational, given that $\sqrt{3}$ is irrational. [CBSE SQP 2020]

73. The LCM of 6^4, 8^2 and k is 12^4 where k is a positive integer. Find the smallest value of k. Show your steps. [CBSE Question Bank 2023]

Ans. Prime factors of $6^4 = (2 \times 3)^4 = 2^4 \times 3^4$

Prime factors of $8^2 = (2^3)^2 = 2^6$

Now, prime factores of $12^4 = (2 \times 2 \times 3)^4$

$= (2^2 \times 3)^4$

$= 2^8 \times 3^4$

So, on comparing the prime factors of 6^4, 8^2 and 12^4, we can see that the smallest values of k is 2^8 or 256.

74. Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number. [CBSE 2020]

75. Three bells toll at intervals of 12 minutes, 15 minutes and 18 minutes respectively. If they start tolling together, after what time will they next toll together? [Diksha]

Ans. The required time is the LCM of 12, 15 and 18.

$\because$ $12 = 2 \times 2 \times 3$

$15 = 3 \times 5$

$18 = 2 \times 3 \times 3$

$\therefore$ LCM $= 2^2 \times 3^2 \times 5 = 180$

So, next time the bells will ring together after 180 minutes or 3 hours.

76. On a morning walk, three people step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk, so that each can cover the same distance in complete steps? [Delhi Gov. QB 2022, CBSE 2015]

77. A merchant has 120 litres and 180 litres of two kinds of oil. He wants to sell oil by filling the two kinds of oil in tins of equal volumes. What is the greatest volume of such a tin?

Ans. In order to find volume of such a tin, we need to find the largest number which exactly divides 120 and 180, which is nothing but the HCF (120, 180).

$120 = 2 \times 2 \times 2 \times 3 \times 5$

$180 = 2 \times 2 \times 3 \times 3 \times 5$

HCF (120, 180) $= 2 \times 2 \times 3 \times 5 = 60$

Hence, the greatest volume of each tin is 60 litres.

78. During a sale, colour pencils were being sold in the pack of 24 each and crayons in the pack of 32 each. If you want full packs of both and the same number of pencils and crayons, how many packets of each would you need to buy?

[Delhi Gov. QB 2022]

Ans. Prime factorisations are:

$24 = 2 \times 2 \times 2 \times 3$

$32 = 2 \times 2 \times 2 \times 2 \times 2$

LCM (24, 32) $= 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$= 96$

According to the question, full packs and same number of pencils and crayons are bought. i.e., 96.

Thus, 96 crayons or $\frac{96}{32} = 3$ packs of crayons

96 pencils of $\frac{96}{24} = 4$ packs of pencils.

79. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p and q are primes. **[NCERT]**

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

80. Prove that $\sqrt{n}$ is not a rational number, if n is not perfect square.

Ans. Let $\sqrt{n}$ be a rational number.

$\therefore \sqrt{n} = \frac{p}{q}$, where p and q are co-prime integers, $q \neq 0$.

On squaring both sides, we get

$$n = \frac{p^2}{q^2}$$

$\Rightarrow p^2 = nq^2$ ...(i)

$\Rightarrow n$ divides p^2

$\Rightarrow n$ divides p ...(ii)

[Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer]

Let $p = nm$, where m is any integer.

$\Rightarrow p^2 = n^2 m^2$

$\Rightarrow nq^2 = n^2 m^2$ [Using (i)]

$\Rightarrow q^2 = nm^2$

$\Rightarrow n$ divides q^2

$\Rightarrow n$ divides q ...(iii)

[Let p be a prime number. If p divides q^2, then p divides q, where q is a positive integer]

From (ii) and (iii), n is a common factor of both p and q which contradicts the assumption that p and q are co-prime integers.

So, our supposition is wrong.

Hence, $\sqrt{n}$ is an irrational number.

81. Prove that $\frac{\sqrt{5}}{7}$ is an irrational number.